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Upper bounds on supersymmetry breaking from gauge coupling unification

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ABSTRACT

I derive conservative upper bounds on the supersymmetry breaking parameter $m_{1/2}$ as a function of the strong coupling in the Standard Supersymmetric Model (SSM) using gauge coupling unification. I find that over more than 99% of the parameter space, $\alpha_3 > 0.120$ implies that $m_{1/2}$ is below 10 TeV and $\alpha_3 > 0.129$ implies that $m_{1/2}$ is below 1 TeV. I express the variation of these bounds over the SSM parameter space with a numerical coefficient, c. I also find that in the SSM, a reasonable value of $50\,\mathrm{GeV} < m_{1/2} < 1\,\mathrm{TeV}$ requires $\alpha_3 > 0.119$ over the whole parameter space. These bounds are particularly sensitive to the value of $\sin^2\theta_W = 0.2317 \pm 0.0005$ used in the calculation. In more realistic models, heavy thresholds and gravitational effects will modify this result. Although these effects are theoretically calculable in specific models, more realistic models contain many unknown parameters in practice. I illustrate this point with minimal supersymmetric $SU(5)$ where the combined constraints of gauge coupling unification and proton decay require $\alpha_3 > 0.119$ for $m_{1/2} < 1\,\mathrm{TeV}$ and the upper bound on the supersymmetry breaking scale is greatly relaxed.

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Gauge coupling unification [1] applied to precision LEP measurements has provided strong motivation for supersymmetry, [2] and inspired numerous attempts to extract the supersymmetry breaking scale and constrain the parameter space of specific models [3]. However, definite conclusions only result from very specific models: threshold effects and gravitational corrections make model-independent statements difficult [4]. In this paper, I find interesting constraints from gauge coupling unification within the specific framework of the Standard Supersymmetric Model (SSM) as defined in reference [5] and the minimal supersymmetric $SU(5)$ model. Although these models are an excellent beginning, they can hardly be considered ultimate theories as neither include gravity and the fine-tuning problem in minimal supersymmetric $SU(5)$ requires some modification of the GUT structure. However, there is hope that similar constraints could be derived for specific realistic string models where the additional threshold and gravitational effects are in principle calculable.

Direct searches for supersymmetric particles have continually increased the lower bounds on their masses. However, the only upper bounds on supersymmetric masses come from naturalness arguments [6] or cosmological constraints on the LSP relic density [7]. Although compelling, the naturalness bounds are not rigorous, and whether the upper bound on supersymmetric masses is 1 TeV, 10 TeV or even 100 TeV is not clear and somewhat a matter of taste. The cosmological bounds can be evaded, for example by breaking R-parity. It would be extremely useful to have some other method of bounding the supersymmetric masses. In this paper, I focus on a first step in this direction by deriving an upper bound on the soft supersymmetry breaking parameter $m_{1/2}$ as a function of the strong coupling in the Standard Supersymmetric Model (SSM). Our approach attempts a general and analytic analysis to reveal the underlying physics, and is meant to complement the many numerical searches of the SSM parameter space in the literature [3]. Unless otherwise indicated, our notation assumes all gauge couplings are renormalized at m_Z in the $\overline{MS}$ scheme.

Ignoring gravitational effects, gauge coupling unification in the SSM gives a simple prediction for the soft supersymmetry breaking parameter $m_{1/2}$. Restricting attention to values of $m_{1/2}$ for which all the supersymmetric thresholds are above m_Z, this prediction is [8]:

$$\ln(\frac{m_{1/2}}{m_Z}) = -X + \frac{7\pi}{\alpha_3} + 7\ln\left[\frac{\alpha_3(m_{\tilde{g}})}{\alpha_2(m_{\tilde{w}})}\right] - \ln(c_{\tilde{w}}) \tag{1}$$

where the gaugino masses are given by

$$m_{\tilde{i}} = c_{\tilde{i}}\ m_{1/2} = \frac{\alpha_i(m_{\tilde{i}})}{\alpha_G} m_{1/2}\ , \tag{2}$$

α_G is the unified coupling at the unification scale, and $f(y,w)$ includes the threshold effects of squarks and sleptons under simple assumptions for the form of the stop mass matrix [8]

$$f(y,w) \;\; = \;\; \frac{15}{8}\ln(\sqrt{c_{\tilde{q}}+y}) - \frac{9}{4}\ln(\sqrt{c_{\tilde{e}_l}+y}) + \frac{3}{2}\ln(\sqrt{c_{\tilde{e}_r}+y})$$

$$-\frac{19}{48}\ln(\sqrt{c_{\tilde{q}}+y+w})-\frac{35}{48}\ln(\sqrt{c_{\tilde{q}}+y-w})\ , \tag{3}$$

where $y \equiv m_0^2/m_{1/2}^2$ and $w \equiv \bar{m}^2/m_{1/2}^2$ with $\bar{m}^2$ representing the off-diagonal elements in the stop mass matrix. The quantity X is given by

$$\begin{aligned} X &= -\frac{15\pi}{\alpha_{em}}\left[.2+\delta_s(gauge)+\delta_s(Yukawa)-\sin^2\theta_W\right] \\ &\quad -f(y,w)+\frac{9}{4}\ln(\frac{m_t}{m_Z})+3\ln(\frac{\mu}{m_Z})+\frac{3}{4}\ln(\frac{m_H}{m_Z})\ , \end{aligned} \tag{4}$$

where μ is the dimensionful higgs mixing term in the SSM superpotential and m_H represents the mass of the charged, pseudoscalar and heavy scalar higgs which are nearly degenerate in the SSM. The gauge numeric corrections to two loop accuracy are [9]

$$\delta_s(gauge) = 0.00127 + 0.01480\alpha_3\ , \tag{5}$$

and numerical calculations give $-0.0004 < \delta_s(Yukawa) < 0$ [10]. To derive the most conservative upper bound on $m_{1/2}$, I will want to maximize the right side of Equation (1), and therefore minimize X. Note, by definition, there are no GUT or intermediate thresholds in the SSM [5].

The calculation is most sensitive to $\sin^2\theta_W$, for which the central value has come down in the last year to [11]

$$\sin^2\theta_W = 0.2317 \pm 0.0005\ . \tag{6}$$

Bounds on other parameters I will use are:

$$\alpha_{em} = \frac{1}{127.9 \pm .0.1} \qquad 131\,\mathrm{GeV} < m_t < 190\,\mathrm{GeV}\ . \tag{7}$$

Although the most conservative choice for μ and m_H would be to take them below m_Z, values of these parameters in the SSM turn out to be nearly proportional to $m_{1/2}$ because of the correlations introduced by radiative electroweak symmetry breaking. In fact, μ and m_H are determined as a function of the five parameters of the SSM: $m_t, \tan\beta, m_{1/2}, \xi_0, \xi_A$. The explicit form of the tree level expressions for μ and m_H show that they both approximately scale with $m_{1/2}$:

$$m_{\mu,H}^2 = a_{\mu,H} m_{1/2}^2 + b_{\mu,H} m_W^2\ . \tag{8}$$

I use a parameter, c, to encode the dependence on μ and m_H in a simple way by the equation

$$\frac{15}{4}\ln(\frac{c\ m_{1/2}}{m_Z}) \equiv 3\ln(\frac{\mu}{m_Z})+\frac{3}{4}\ln(\frac{m_H}{m_Z})\ . \tag{9}$$

Note that throughout this paper, the logs associated with a particular threshold correction are understood to be zero if the threshold is below m_Z. A Monte Carlo search of parameter space reveals that $c > 0.5$ for more than 99% of the points

considered. Our search uses $m_{1/2} = 1\,\mathrm{TeV}$ and randomly selects $10,000$ points for each sign of μ over the four dimensional parameter space defined by $130\,\mathrm{GeV} < m_t < 190\,\mathrm{GeV}$, $1 < \tan\beta < 50$, $-10 < \xi_0 < 10$, and $-2|\xi_0| < \xi_A < 2|\xi_0|$. The parameters m_t, ξ_0, ξ_A are searched on a linear scale while the parameter $\tan\beta$ is searched on a logarithmic scale.

To reliably extract $m_{1/2}$, the dependence of the gaugino masses on $m_{1/2}$ (the EGM effect) must be carefully taken into account [12]. To derive the most conservative upper bound on $m_{1/2}$, I must use an upper bound on $\alpha_3(m_{\tilde{g}})$, and and a lower bound on $\alpha_2(m_{\tilde{w}})$. The simplest way to extract the gauge couplings renormalized at the gaugino mass is to iterate the expression

$$\alpha_i(m_{\tilde{i}}) = \frac{\alpha_i}{1 - \alpha_i \frac{b_i}{2\pi} \ln(\frac{\alpha_i(m_{\tilde{i}}) m_{1/2}}{\alpha_G m_Z})} \tag{10}$$

along with the expression for the gaugino mass, Equation (2), to a solution. Since these values for the gauge couplings increase with b_i, I take the maximum value of b_3 below the gluino threshold and the minimum value of b_2 below the wino threshold to derive the most conservative bounds. Since in the SSM, the squarks cannot be much lighter than the gluino, I use $b_3 = -7$. Below the wino threshold, I use the minimum $b_2 = -19/6$. Numerical calculations show that an upper bound on α_G gives the most conservative bound for $m_{1/2}$. I obtain this upper bound on α_G by extrapolating the hypercharge coupling $\alpha_y(m_Z)$ to the scale $10^{17}\,\mathrm{GeV}$ using the maximum $b_y = 33/5$. This results in $\alpha_G < 0.0454$ after numerically correcting for two-loop effects.

Finding an upper bound on $f(y,w)$ depends on the physically acceptable range of the variables y and w. Although the minimum value of $f(y,w) = -0.025$ has already been determined [13], the maximum value is not as clear cut. For fixed w, $f(y,w)$ approaches zero as y becomes very large. Maximization with respect to values of w yielding positive $m_{\tilde{t}}^2$ gives a maximum of $f(y,w) = 35 ln(\sqrt{y})/48$ at $w = c_{\tilde{q}} + y$ as y becomes very large. However, a more physically reasonable choice is $|w| < d m_t/m_{1/2}$ where d is a positive numeric coefficient of order one. Using this form for w, I have numerically scanned over values of $d < 10$ to determine that, for $m_{1/2} > 1\,\mathrm{TeV}$, $f(y,w) < 0.5$, which I will use in the calculation. It is the fact that the $SU(3), SU(2), U(1)$ beta functions are all equal for an entire generation coupled with the assumption of universal soft supersymmetry breaking which severely restricts the range of $f(y,w)$ and makes the results of gauge coupling unification approximately independent of the universal scalar mass m_0.

Putting all this together gives equations which can be iterated to find an upper bound on $m_{1/2}$ as a function of the strong coupling. The choice of parameters I make to minimize X and thereby obtain a conservative bound are:

$$\alpha_{em} = \frac{1}{127.8} \quad m_t = 131\,\mathrm{GeV} \quad f(y,w) = 0.5 \quad \alpha_G = 0.0454\ . \tag{11}$$

The bound using $c = 0.5$ is plotted as solid lines in Figure 1 for the central and $1-\sigma$ values of $\sin^2\theta_W$. If $\alpha_3 > 0.120$, $m_{1/2}$ is constrained to be less than $10\,\mathrm{TeV}$ and if

$\alpha_3 > 0.129$, $m_{1/2}$ is constrained to be less than 1 TeV. These solid lines would move downward as c increases giving tighter bounds on $m_{1/2}$ and would move upward as c decreases giving looser bounds on $m_{1/2}$. To quantify this effect, in Figure 2 I indicate the value of α_3, as a function of c, above which $m_{1/2}$ is bounded by $1\,\mathrm{TeV}, 10\,\mathrm{TeV}$ with solid,dashed lines. In our Monte Carlo, among the 12,231 out of 20,000 points which have perturbative Yukawas and a stable electoweak breaking minimum, the values of c are distributed as follows: 62.9% give $c > 2$, 29.9% give $2 > c > 1$, 6.4% give $1 > c > 0.5$, 0.7% give $0.5 > c > 0.2$, and 0.1% give $c < 0.2$. The points with $c < 0.5$ all have $\mu/m_{1/2} < 0.4$ but rarely have $m_H/m_{1/2} < 1$. This observation and the form of Equation (9) indicate that the regions with small c are the regions where $\mu/m_{1/2}$ is fine-tuned to be small.

Previous work pursued an alternative to finding an upper bound on the supersymmetry breaking scale and derived a lower bound on α_3 as a function of $m_{1/2}$ [13]. For values of $m_{1/2}$ giving wino masses above m_Z this derivation followed from Equation (1). For values of $m_{1/2}$ giving wino masses below m_Z, the relation becomes

$$\ln(\frac{m_{1/2}}{m_Z}) = \frac{1}{7}X - \frac{\pi}{\alpha_3} - \ln(c_{\tilde{g}}) \ . \tag{12}$$

However, for both cases, the most conservative bound on α_3 results from maximizing X, minimizing $c_{\tilde{g}}$ and maximizing $c_{\tilde{w}}$. This is accomplished by taking $b_3 = -7$, $b_2 = -1/3$, and

$$\alpha_{em} = \frac{1}{128.0} \qquad m_t = 190\,\mathrm{GeV} \qquad f(y,w) = -0.025 \qquad \mu = m_H = 1\,\mathrm{TeV} \ . \tag{13}$$

I use a minimum value of α_G in Equation (1) and a maximum value of α_G in Equation (12) from the range

$$\frac{3}{20\alpha_{em}} + \frac{3}{5\alpha_3} - 0.7 < \frac{1}{\alpha_G} < \frac{3}{20\alpha_{em}} + \frac{3}{5\alpha_3} + 1.4 \tag{14}$$

obtained from gauge coupling unification with a 3 TeV bound on the supersymmetric thresholds.

The resulting bound on α_3 is shown for the central and $1-\sigma$ values of $\sin^2\theta_W$ by wavy lines in Figure 1. To have $50\,\mathrm{GeV} < m_{1/2} < 1\,\mathrm{TeV}$ requires $\alpha_3 > 0.119$. Note the differences in the approaches leading to the upper bound on $m_{1/2}$ indicated by solid lines and the lower bound on α_3 indicated by wavy lines. The upper bound on $m_{1/2}$ is obtained by maximizing the right side of Equation (1) with no bounds on the supersymmetric thresholds. The lower bound on α_3 is obtained by minimizing the right side of Equation (1) and maximizing the right side of Equation (12) assuming that $m_{1/2}, \mu, m_H < 1\,\mathrm{TeV}$. The ultimate reason for this difference is that the supersymmetric threshold corrections to $\sin^2\theta_W$ in the SSM are bounded from above by the assumption of universal soft supersymmetry breaking but are only bounded from below by naturalness.

As a simple example of modifications required in more realistic models, consider minimal supersymmetric $SU(5)$. Although the predictions of gauge coupling unification depend on the unknown superheavy scales, one can extract the mass of the superheavy proton decay mediating triplets as a function of the low-energy couplings [14, 15, 9]. Unfortunately, this leaves no prediction for the supersymmetry breaking scale. However, gauge coupling constant unification, a 1 TeV naturalness bound, and limits on the proton lifetime can be combined to derive a lower bound on α_3 [9]. Using the value of $\sin^2\theta_W$ in Equation (6) gives a bound of $\alpha_3 > 0.119$ in minimal supersymmteric $SU(5)$. Careful study [16] reveals that in minimal supersymmetric $SU(5)$, the supersymmetry breaking scale could be as large as $10^8\,\mathrm{GeV}$ and extensions of the minimal model further relax this bound.

In addition to the theoretical prejudice for a naturalness bound on supersymmetry, this calculation offers the possibility of using coupling constant unification to place a bound on $m_{1/2}$ in specific models. The excellent agreement of gauge coupling unification with the SSM, combined with the sharpening and shifting measurements of the low-energy couplings provide interesting speculation. On the one hand, apart from the highly unlikely possibility of a light gluino [17], measurements of the strong coupling from deep-inelastic scattering [18], charmonium [19], and Υ [20] give low values of α_3 which in the SSM and minimal supersymmetric $SU(5)$ require unnaturally high values of $m_{1/2}$. On the other hand, the values of α_3 from jet shapes at LEP are increasing to values which ensure a low value of $m_{1/2}$ in the SSM. To complicate matters, the value of $\sin^2\theta_W$ from SLD [21] is lower than that from LEP introducing more uncertainty in this critical input to gauge coupling unification calculations.

Even though these types of conclusions can so far only be reached in very simple models like the SSM and minimal supersymmetric $SU(5)$, the dependence of the results on the unfolding experimental situation holds for some the excitement of a close horse race. The observation that low-energy limits of string models come very close to the simple models considered in this paper, and the ability of string theory to quantify gravitational corrections to the predictions of gauge coupling unification, offers hope of testing the physics of unified theories using precision low-energy measurements.

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Figure Captions

Figure 1: Solid lines indicate the upper bound on the supersymmetry breaking scale from gauge coupling unification in the SSM for central and $1-\sigma$ values of $sin^2\theta = 0.2317 \pm 0.0005$ with c=0.5. The supersymmetry breaking parameter $m_{1/2}$ is below $10\,\mathrm{TeV}$ for $\alpha_3 > 0.120$ and is below $1\,\mathrm{TeV}$ for $\alpha_3 > 0.129$. Wavy lines indicate a lower bound on the strong coupling from gauge coupling unification assuming a naturalness bound on the scale of superysmmetry breaking. Reasonable values of $50\,\mathrm{GeV} < m_{1/2} < 1\,\mathrm{TeV}$ require $\alpha_3 > 0.119$.

Figure 2: The value of α_3, as a function of c, above which $m_{1/2}$ is bounded by $1\,\mathrm{TeV}, 10\,\mathrm{TeV}$ is indicated by solid,dashed lines in the SSM for central and $1-\sigma$ values of $sin^2\theta = 0.2317 \pm 0.0005$. The most conservative choice, $\mu, m_H < m_Z$ corresponds to $c = 0$. A Monte Carlo search of the SSM parameter space finds that over 99% of the points considered give $c > 0.5$.